THE TRANSFER OF HEAT IN THE LAMINAR FLOW OF AN INCOMPRESSIBLE LIQUID IN AN ANNULAR CHANNEL WITH NONSYMMETRIC BOUNDARY CONDITIONS OF THE II-ND KIND RELATIVE TO THE AXIS OF THE FLOW

L. I. Urbanovich

Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 2, pp. 326–328, 1968

UDC 536.25:532.517.2

We have derived expressions for the distribution of temperatures over the cross section and for the Nusselt numbers on the two walls of an annular channel for the case of developed laminar flow of an incompressible liquid with a constant heat-flux density along the length, but variable over the perimeter of the outside wall.

Let us consider the problem of the transfer of heat in the laminar flow of a liquid through a tube of annular cross section. We will adopt the following assumptions: 1) the flow of the liquid is laminar and steady-state; 2) the physical properties of the liquid are constant; the liquid itself is incompressible; 3) the motion of the liquid has been stabilized hydrodynamically and thermally; 4) there are no internal heat sources in the liquid; 5) the heat-flux density at the inside wall of the channel is constant along the length and over the perimeter: $q_1 = \text{const}$; 6) the heat flux density at the outside wall is constant over the length, but variable over the perimeter: $q_2 = 0$ when $\delta < \varphi < 2\pi - \delta$ and $q_2 = q_0 (1 + m \cos\varphi)$ when $-\delta \le \varphi \le \delta$. Here $q_0 = \text{const}$, $m = \text{const}$. Having solved the equations of motion and energy for the indicate conditions, we obtained the following results:

$$\begin{aligned}
\Theta = {} & K_1\left[\frac{R^2}{4} - \frac{R^4}{16} + \frac{R_2^2-1}{4\ln R_2} R^2(\ln R - 1)\right] \\
& - K_2\left[\frac{(R_2^2-1)^2}{8\ln^2 R_2}\ln^2 R + \frac{R^4}{16} - \frac{R_2^2-1}{4\ln R_2}R^2\right] \\
& + \sum_{n=2}^{\infty}\frac{q_{w0}r_2}{\lambda}\,\frac{1}{\pi R_2^{n-1}}\left[\frac{2\sin n\delta}{n} + \frac{m\sin(n+1)\delta}{n+1} + \frac{m\sin(n-1)\delta}{n-1}\right] \times\left(R^n\frac{\cos n\varphi}{n} - R\cos\varphi\right) \\
& + \left\{\frac{q_{w1}r_1}{\lambda} + \frac{K_2}{4}\left[1 - \frac{2(R_2^2-1)}{\ln R_2}\right] - \frac{K_1}{4}\left(1 - \frac{R_2^2-1}{\ln R_2}\right)\right\} R\cos\varphi \\
& + \Bigg\{K_1\Big[50R_2^8 - 21R_2^8\ln R_2 - 50R_2^6 - 50R_2^2 + 21\ln R_2 + 50 + \frac{27}{\ln R_2}(1 - 2R_2^2 + 2R_2^6 - R_2^8)\Big] \\
& + K_2\Big[-182R_2^8 + 470R_2^6 - 396R_2^4 + 110R_2^2 - 2 + 12\ln R_2(7R_2^8 - 19R_2^6 + 18R_2^4 - 6R_2^2) \\
& + \frac{18}{\ln R_2}(7R_2^8 - 20R_2^6 + 18R_2^4 - 4R_2^2 - 1) - \frac{54}{\ln^2 R_2}(R_2^8 - 4R_2^6 + 6R_2^4 - 4R_2^2 + 1)\Big]\Bigg\} \\
& \times\left[288(R_2^4\ln R_2 - \ln R_2 - R_2^4 + 2R_2^2 - 1)\right]^{-1}. \qquad (1)
\end{aligned}$$

If we know the temperature field of the liquid, we can easily determine the Nusselt numbers for the two sides of the tube. Thus, for the outside wall we have

$$\begin{aligned}
\frac{1}{\mathrm{Nu}_2(\varphi)} = {} & \frac{1}{8(1+m\cos\varphi)}\left(\frac{1}{R_2}\frac{q_{w1}}{q_{w0}} + \frac{\delta + m\sin\delta}{\pi}\right) \\
& \times\Bigg\{\left[(50R_2^8 - 21R_2^8\ln R_2 - 50R_2^6 - 50R_2^2 + 21\ln R_2 + 50)\ln R_2 + 27(1 - 2R_2^2 + 2R_2^6 - R_2^8)\right] \\
& \times\left\{18\left[(R_2^4-1)\ln R_2 - (R_2^2-1)^2\right]^2\right\}^{-1} + \frac{3R_2^4\ln R_2 - 4R_2^2(R_2^2-1)}{(R_2^4-1)\ln R_2 - (R_2^2-1)^2}\Bigg\} \\
& + \frac{\Delta p^2 r_1^3}{128\mu q_{w0} l^2 R_2(1+m\cos\varphi)} \times\Bigg\{\frac{4R_2^2(R_2^2-1)}{\ln R_2} - 3R_2^4 + 4R_2^2 - 2 \\
& + \Big[-182R_2^8 + 470R_2^6 - 396R_2^4 + 110R_2^2 - 2 + 12\ln R_2(7R_2^8 - 19R_2^6 + 18R_2^4 - 6R_2^2) \\
& + \frac{18}{\ln R_2}(7R_2^8 - 20R_2^6 + 18R_2^4 - 4R_2^2 - 1) - \frac{54}{\ln^2 R_2}(R_2^8 - 4R_2^6 \\
& + 6R_2^4 - 4R_2^2 + 1)\Big] \Big/ 18\left[(R_2^4-1)\ln R_2 - (R_2^2-1)^2\right]\Bigg\} \\
& + \sum_{n=2}^{\infty}\frac{1}{2\pi(1+m\cos\varphi)}\left[\frac{2\sin n\delta}{n} + \frac{m\sin(n+1)\delta}{n+1} + \frac{m\sin(n-1)\delta}{n-1}\right]\left(R_2\frac{\cos n\varphi}{n} - R_2^{2-n}\cos\varphi\right) \\
& + \frac{\cos\varphi}{2(1+m\cos\varphi)}\Bigg\{\frac{q_{w1}}{q_{w0}} + \frac{\Delta p^2 r_1^3}{16\mu q_{w0} l^2}\left[1 - \frac{2(R_2^2-1)}{\ln R_2}\right] \\
& - \left[\frac{q_{w1}}{q_{w0}} + \frac{R_2(\delta + m\sin\delta)}{\pi}\right]\frac{\ln R_2 + 1 - R_2^2}{(R_2^4-1)\ln R_2 - (R_2^2-1)^2}\Bigg\}. \qquad (2)
\end{aligned}$$

From the expressions for Nu_1 and Nu_2 we can derive several particular solutions. For example, when $m = 0$ we have a case in which the segment of the outside surface with the angle 2δ is heated at a constant flux density $q_{w2} = q_{w0}$, while the remaining portion of the surface is thermally insulated. (The conditions for the heating of the inside wall of the tube remain unchanged.) Finally, if we assume that the ouside wall is heated when

$$q_{w0} = \text{const} \quad (\varphi = \pi, \ m = 0)$$

and energy dissipation is neglected, as

$$R_2 \to \infty \ (r_1 \to 0) \quad \text{and} \ q_{w1} = 0$$

From (2) we obtain $1/Nu_2 = 1/8(3-21/8) = 11/48$, or $Nu_2 = 4.36$ is a known value when $q_{w0} = \text{const}$ in the case of a round tube.

NOTATION

x is the coordinate along the flow; φ is the azimuthal coordinate; δ is the fixed value of φ; r is the radial coordinate; r_1 and r_2 are the radii of internal and external channel surfaces, respectively; $R = r/r_1$ is the dimensionless radius; Δp is the pressure drop across the channel length l; $\Theta = t(r, \varphi, x) - \bar{t}(x)$ is the temperature of the liquid with respect to the means; t is the temperature of the liquid; $\bar{t}$ is the mean temperature of the liquid; q_1 and q_2 are the heat-flux densities on the internal and external walls of channels, respectively; μ is the dynamic viscosity of the liquid; λ is the thermal conductivity of the liquid;

$$K_1 = \frac{4\left[q_{c1}\pi r_1 + q_{c0} r_2 (\delta + m \sin \delta)\right]}{\pi\lambda \left[R_2^4 - 1 - \frac{(R^2-1)^2}{\ln R_2}\right]} \quad \text{and} \quad K_2 = \frac{\Delta p r_1^4}{4\mu\lambda l^2}$$

are the dimensional constant numbers.

REFERENCES

1. B. S. Petukhov, Heat Transfer and Resistance in the Laminar Flow of Liquids through Tubes [in Russian], Energiya, 1967

7 December 1967

Lipetsk Branch, Moscow Institute of Steel and Alloys